

## Standing Wave Patterns

In practical applications it is very convenient to plot the **magnitude** of **phasor voltage** and **phasor current** along the transmission line. These are the **standing wave patterns**:

$$\text{Loss - less line} \left\{ \begin{array}{l} |V(d)| = |V^+| \cdot |1 + \Gamma(d)| \\ |I(d)| = \left| \frac{V^+}{Z_0} \right| \cdot |1 - \Gamma(d)| \end{array} \right.$$

$$\text{Lossy line} \left\{ \begin{array}{l} |V(d)| = |V^+ e^{\alpha d}| \cdot |1 + \Gamma(d)| \\ |I(d)| = \left| \frac{V^+ e^{\alpha d}}{Z_0} \right| \cdot |1 - \Gamma(d)| \end{array} \right.$$

The **standing wave patterns** provide the top envelopes that bound the time-oscillations of voltage and current along the line. In other words, **the standing wave patterns provide the maximum values that voltage and current can ever establish at each location of the transmission line** for given load and generator, due to the interference of incident and refelected wave.

The patterns present a succession of **maxima** and **minima** which repeat in space with a **period** of length  $\lambda/2$ , due to constructive or destructive interference between forward and reflected waves. The patterns for a loss-less line are exactly **periodic** in space, repeating with a  $\lambda/2$  **period**.

Again, note that although we talk about **maxima** and **minima** of the standing wave pattern we are always examining a **maximum** of voltage or current that can be achieved at a transmission line location during any period of oscillation.

We limit now our discussion to the **loss-less** transmission line case where the **generalized reflection coefficient** varies as

$$\Gamma(\mathbf{d}) = \Gamma_R \exp(-j2\beta \mathbf{d}) = |\Gamma_R| \exp(j\phi) \exp(-j2\beta \mathbf{d})$$

Note that the magnitude of an **exponential** with **imaginary argument** is always **unity**

$$|\exp(j\phi) \exp(-j2\beta \mathbf{d})| = 1$$

In a loss-less line it is **always** true that, for any line location,

$$|\Gamma(\mathbf{d})| = |\Gamma_R|$$

When **d** increases, moving from load to generator, the **generalized reflection coefficient** on the complex plane moves **clockwise** on a circle with radius  **$|\Gamma_R|$**  and is identified by the angle  **$\phi - 2\beta \mathbf{d}$** .

The **voltage standing wave pattern** has a **maximum** at locations where the generalized reflection coefficient is **real and positive**

$$\Gamma(\mathbf{d}) = |\Gamma_R|$$

$$\exp(j\phi) \exp(-j2\beta \mathbf{d}) = 1 \quad \Rightarrow \quad |\phi - 2\beta \mathbf{d}| = 2n\pi$$

At these locations we have

$$|1 + \Gamma(\mathbf{d})| = 1 + |\Gamma_R|$$

$$\Rightarrow V_{\max} = |V(\mathbf{d}_{\max})| = |V^+| \cdot (1 + |\Gamma_R|)$$

The phase angle  $\phi - 2\beta \mathbf{d}$  changes by an amount  $2\pi$ , when moving from one maximum to the next. This corresponds to a distance between successive maxima of  $\lambda/2$ .

The **voltage standing wave pattern** has a **minimum** at locations where the generalized reflection coefficient is **real and negative**

$$\Gamma(\mathbf{d}) = -|\Gamma_R|$$

$$\exp(j\phi) \exp(-j2\beta \mathbf{d}) = -1 \quad \Rightarrow \quad |\phi - 2\beta \mathbf{d}| = (2n + 1)\pi$$

At these locations we have

$$|1 + \Gamma(\mathbf{d})| = 1 - |\Gamma_R|$$

$$\Rightarrow V_{\min} = |V(\mathbf{d}_{\min})| = |V^+| \cdot (1 - |\Gamma_R|)$$

Also when moving from one minimum to the next, the phase angle  $\phi - 2\beta \mathbf{d}$  changes by an amount  $2\pi$ . This again corresponds to a distance between successive minima of  $\lambda/2$ .

The voltage standing wave pattern provides immediate information on the transmission line circuit

- ❑ If the load is matched to the transmission line ( $Z_R = Z_0$ ) the voltage standing wave pattern is flat, with value  $|V^+|$ .
- ❑ If the load is real and  $Z_R > Z_0$ , the voltage standing wave pattern starts with a maximum at the load.
- ❑ If the load is real and  $Z_R < Z_0$ , the voltage standing wave pattern starts with a minimum at the load.
- ❑ If the load is complex and  $\text{Im}(Z_R) > 0$  (inductive reactance), the voltage standing wave pattern initially increases when moving from load to generator and reaches a maximum first.
- ❑ If the load is complex and  $\text{Im}(Z_R) < 0$  (capacitive reactance), the voltage standing wave pattern initially decreases when moving from load to generator and reaches a minimum first.

Since in all possible cases

$$|\Gamma(\mathbf{d})| \leq 1$$

the **voltage standing wave pattern**

$$|V(\mathbf{d})| = |V^+| \cdot |(1 + \Gamma(\mathbf{d}))|$$

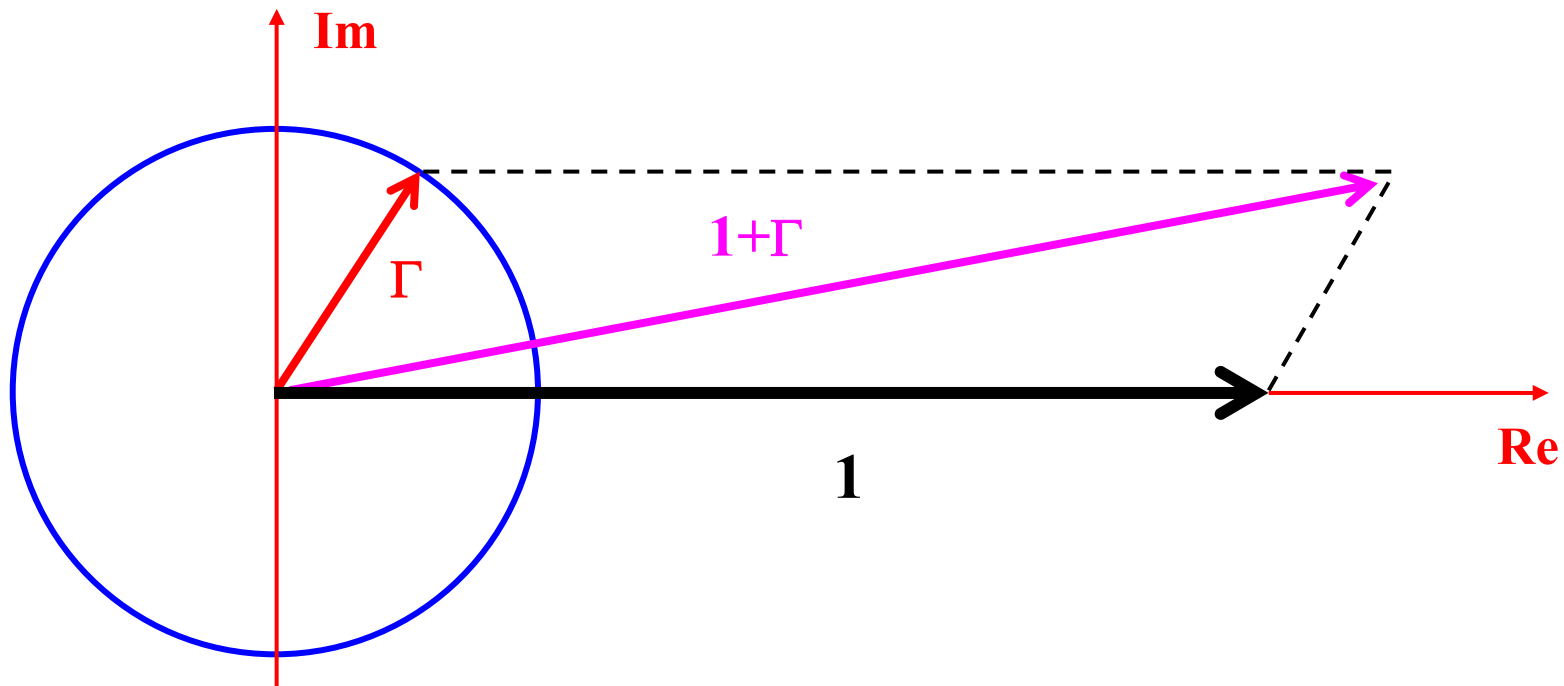
cannot exceed the value  $2 |V^+|$  in a loss-less transmission line. If the load is a short circuit, an open circuit, or a pure reactance, there is total reflection with

$$|\Gamma(\mathbf{d})| = 1$$

since the load cannot consume any power. The voltage standing wave pattern in these cases is characterized by

$$V_{\max} = 2 |V^+| \quad \text{and} \quad V_{\min} = 0 .$$

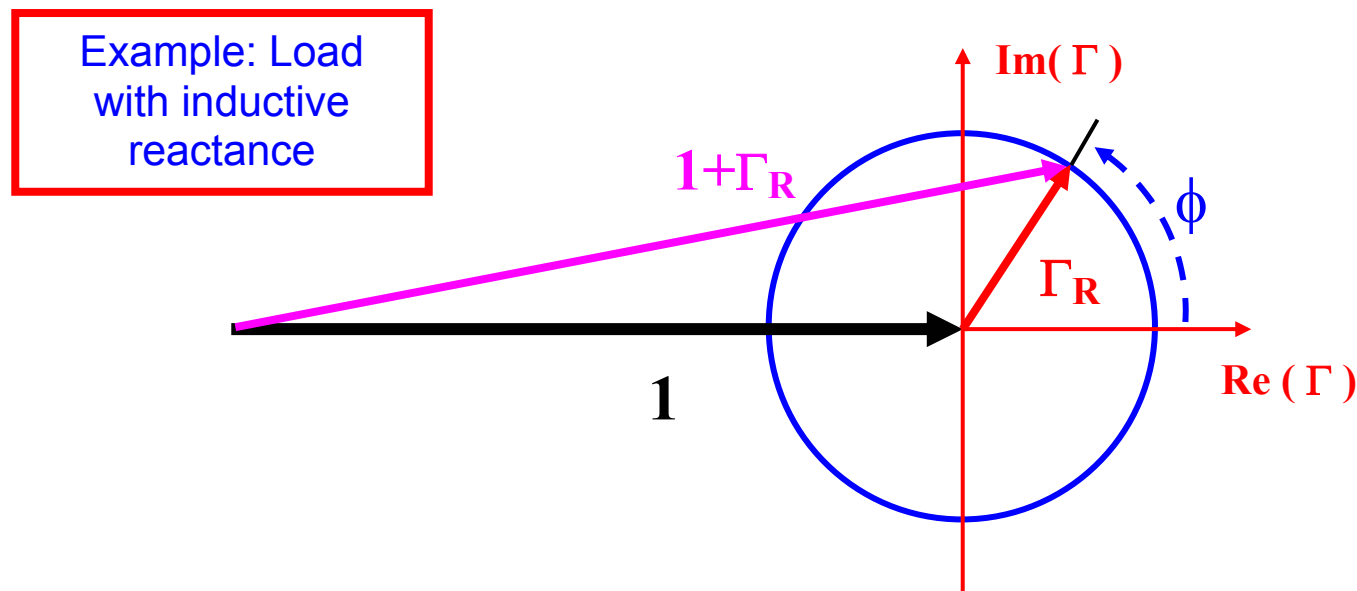
The quantity  $1 + \Gamma(d)$  is in general a complex number, that can be constructed as a vector on the complex plane. The number  $1$  is represented as  $1 + j0$  on the complex plane, and it is just a vector with coordinates  $(1,0)$  positioned on the Real axis. The reflection coefficient  $\Gamma(d)$  is a complex number such that  $|\Gamma(d)| \leq 1$ .

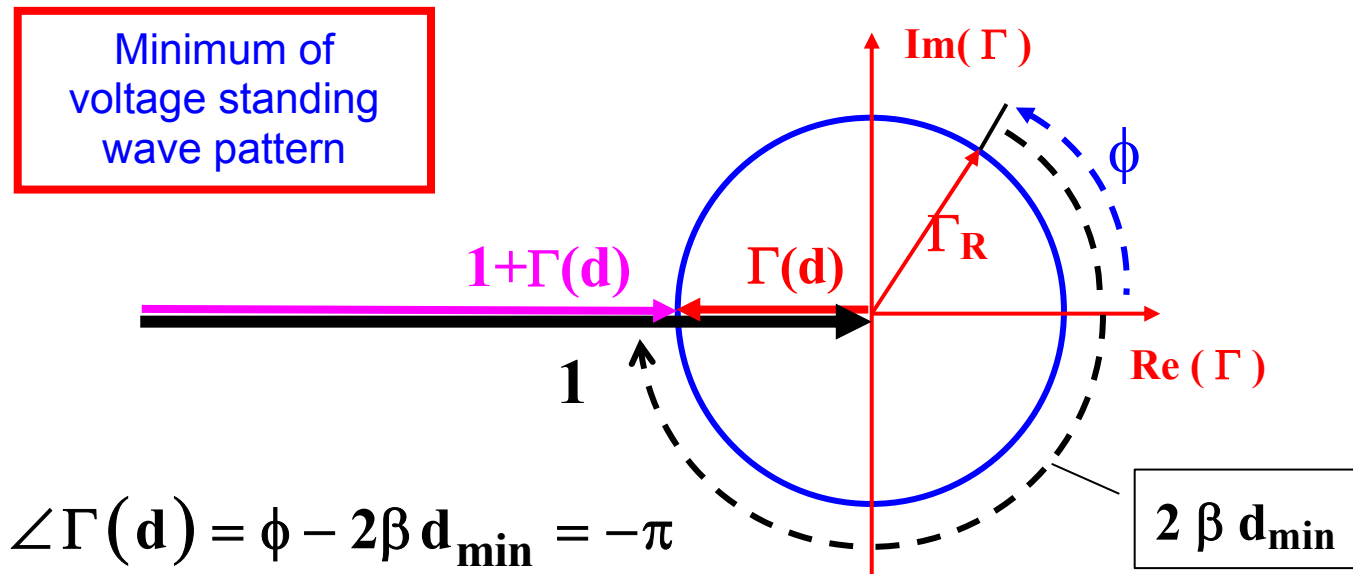
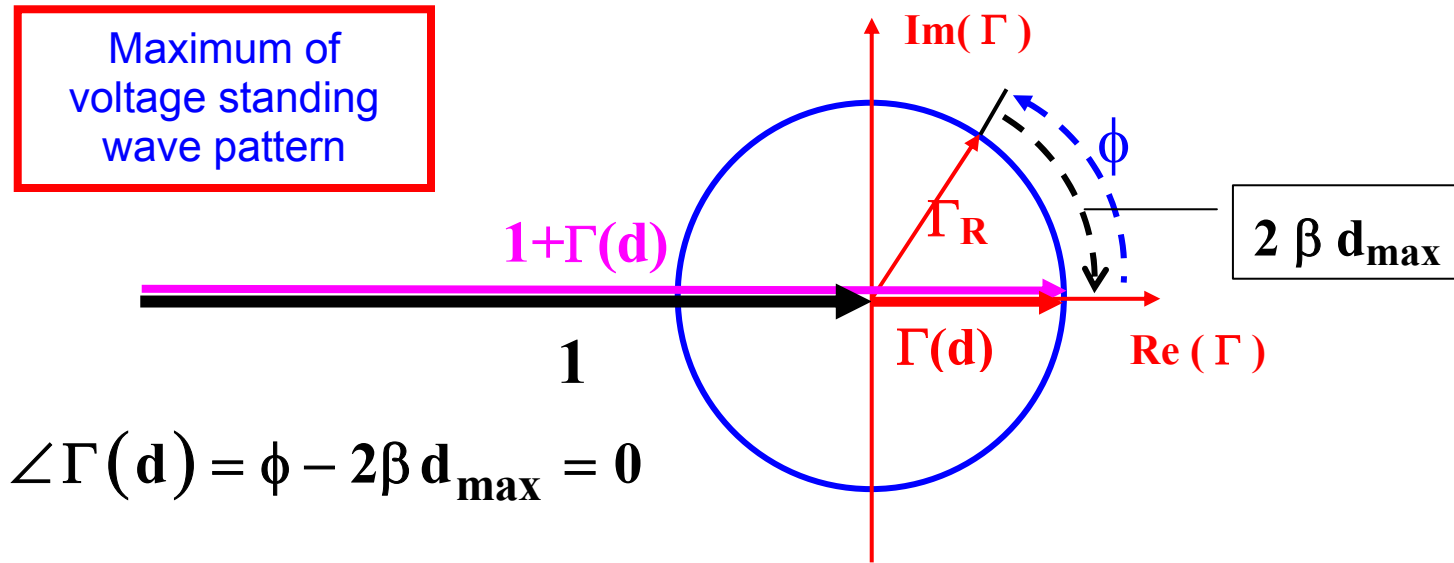


We can use a geometric construction to visualize the behavior of the **voltage standing wave pattern**

$$|V(\mathbf{d})| = |V^+| \cdot |(1 + \Gamma(\mathbf{d}))|$$

simply by looking at a vector plot of  $|1 + \Gamma(\mathbf{d})|$ .  $|V^+|$  is just a scaling factor, fixed by the generator. For convenience, we place the reference of the complex plane representing the reflection coefficient in correspondence of the tip of the vector  $(1, 0)$ .





The **voltage standing wave ratio (VSWR)** is an indicator of load matching which is widely used in engineering applications

$$VSWR = \frac{V_{\max}}{V_{\min}} = \frac{1 + |\Gamma_R|}{1 - |\Gamma_R|}$$

When the load is perfectly **matched** to the transmission line

$$\Gamma_R = 0 \quad \Rightarrow \quad VSWR = 1$$

When the load is **a short circuit, an open circuit or a pure reactance**

$$|\Gamma_R| = 1 \quad \Rightarrow \quad VSWR \rightarrow \infty$$

We have the following useful relation

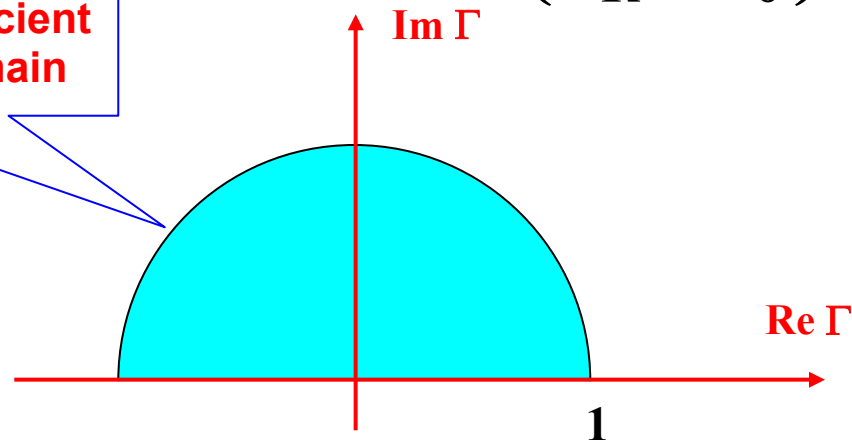
$$|\Gamma_R| = \frac{VSWR - 1}{VSWR + 1}$$

**Maxima** and **minima** of the voltage standing wave pattern.

□ **Load with inductive reactance**

$$\text{Im}(Z_R) > 0 \quad \Rightarrow \quad \text{Im}(\Gamma_R) = \text{Im}\left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) > 0$$

The load reflection coefficient is in this part of the domain

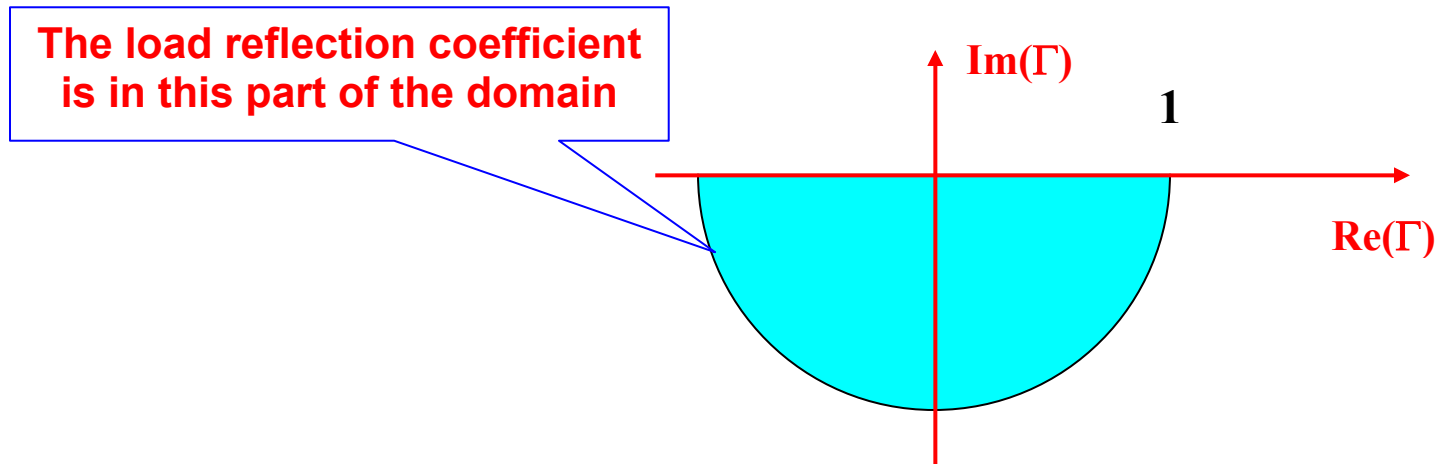


The first maximum of the voltage standing wave pattern is closest to the load, at location

$$\angle \Gamma(d) = \phi - 2\beta d_{\max} = 0 \quad \Rightarrow \quad d_{\max} = \frac{\phi}{4\pi} \lambda$$

□ **Load with capacitive reactance**

$$\text{Im}(Z_R) < 0 \quad \Rightarrow \quad \text{Im}(\Gamma_R) = \text{Im}\left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) < 0$$



The first minimum of the voltage standing wave pattern is closest to the load, at location

$$\angle \Gamma(\mathbf{d}) = |\phi - 2\beta \mathbf{d}_{\min}| = \pi \quad \Rightarrow \quad \mathbf{d}_{\min} = \frac{|\pi - \phi|}{4\pi} \lambda$$

A **measurement** of the **voltage standing wave pattern** provides the locations of the first voltage **maximum** and of the first voltage **minimum** with respect to the load.

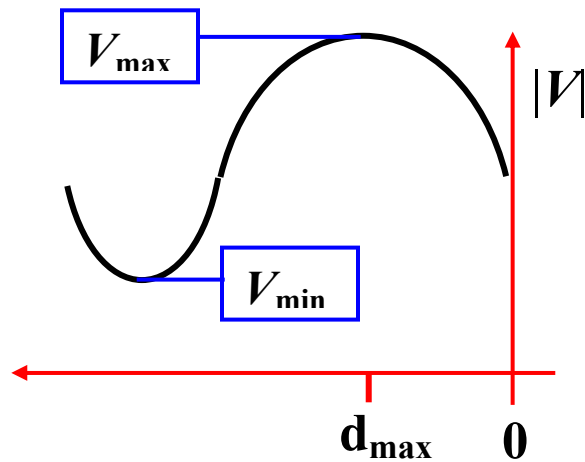
The ratio of the voltage magnitude at these points gives directly the voltage standing wave ratio (VSWR).

This information is sufficient to **determine** the **load impedance**  $Z_R$ , if the **characteristic impedance** of the transmission line  $Z_0$  is known.

- **STEP 1: The VSWR provides the magnitude of the load reflection coefficient**

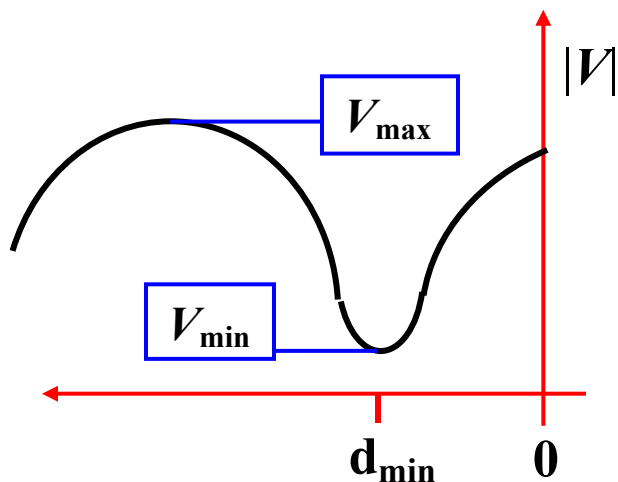
$$|\Gamma_R| = \frac{VSWR - 1}{VSWR + 1}$$

- **STEP 2: The distance from the load of the first maximum or minimum gives the phase  $\phi$  of the load reflection coefficient.**



For an inductive reactance, a voltage maximum is closest to the load and

$$\phi = 2\beta d_{\max} = \frac{4\pi}{\lambda} d_{\max}$$



For a capacitive reactance, a voltage minimum is closest to the load and

$$\phi = -\pi + 2\beta d_{\min} = -\pi + \frac{4\pi}{\lambda} d_{\min}$$

- **STEP 3: The load impedance is obtained by inverting the expression for the reflection coefficient**

$$\Gamma_R = |\Gamma_R| \exp(j\phi) = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$\Rightarrow Z_R = Z_0 \frac{1 + |\Gamma_R| \exp(j\phi)}{1 - |\Gamma_R| \exp(j\phi)}$$