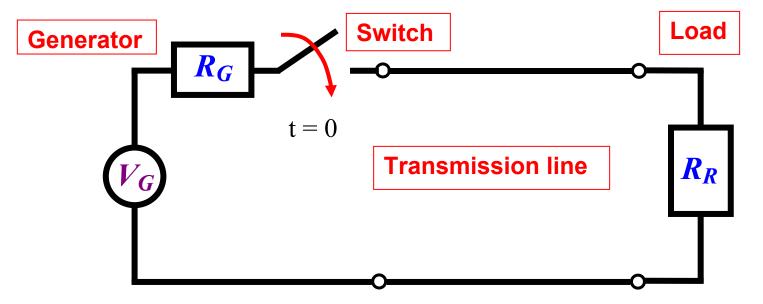# Transient and Steady-State on a Transmission Line

We need to give now a **physical interpretation** of the mathematical results obtained for transmission lines. First of all, note that we are considering a <mark>steady-state regime</mark> where the wave propagation along the transmission line is perfectly periodic in time. This means that all the <mark>transient phenomena</mark> have already decayed.

To give a feeling of what the steady-state regime is, consider a transmission line that is connected to the generator by closing a switch at a reference time **t = 0**. For simplicity we assume that all impedances, including the line characteristic impedance, are real.

After the switch is closed, the voltage at the input of the transmission line will vary nearly **instantaneously** from the <mark>open voltage</mark> of the generator V_G to a value $V+$, with a current $I+$ beginning to flow into the line. A <mark>transient</mark> takes place in the transmission line, as charges in the conductors move, transporting the current towards the load. **Until the front first reaches the end of the transmission line, the load voltage remains zero**.

Initially, the **input impedance** of the transmission line appears to be the same as the line **characteristic impedance**, because the current cannot yet sense the value of the **load impedance**. Therefore, a voltage front $V+$ propagates with the current front $I+$, where

$$V^+ = I^+ \, Z_0 = V_G \frac{Z_0}{R_G + Z_0}$$

$$I^+ = \frac{V^+}{Z_0} = \frac{V_0}{R_G + Z_0}$$

If the load does not match exactly the characteristic impedance of the line, the voltage $V+$ and the current $I+$ cannot be established across the load R_R , when the front reaches the end of the line, because

$$V^+ \neq I^+ R_R$$

Therefore, voltage and current adjust themselves at the load by reflecting back a wave front with voltage V^- and current I^- such that

$$V^+ + V^- = (I^+ + I^-) R_R$$

Since also the reflected front will see an impedance Z_0 we have

$$V^+ = Z_0 \, I^+ \, ; \qquad V^- = -Z_0 \, I^-$$

$$\Rightarrow \quad V^- = V^+ \, \frac{R_R - Z_0}{R_R + Z_0}$$

The quantity

$$\Gamma_R = \frac{V^-}{V^+} = \frac{R_R - Z_0}{R_R + Z_0}$$

is the <mark>load reflection coefficient</mark>.

The wave reflected by the load propagates in the negative direction and interferes with the oscillating values of voltage and current found along the transmission line, which continue to be injected by the generator.

When the reflected wave reaches the input of the transmission line, it terminates on the generator impedance R_G and if this does not match the line characteristic impedance, reflection back into the line again occurs, generating now an additional forward wave with associated voltage

$$V_2^+ = V^- \, \frac{R_G - Z_0}{R_G + Z_0}$$

Remember that the ideal voltage source part of the generator will behave simply as a short for the reflected wave attempting to exit the line from the input.

The front reflected by the generator side will again reach the load, and waves of ever decreasing amplitude will keep bouncing back and forth along the line until the process associated to that initial wave front dies out.

Every subsequent wave front injected over time by the oscillating generator undergoes an identical phenomenon of multiple reflections. If we assume a sinusoidal generator, voltage and current injected into the line repeat periodically, according to the period of the oscillation. Therefore, successive reflections at the ends of the line obey the same reflection coefficients, but involving in time different values of amplitude and phase.

If the generator continues to supply the line with a stable oscillation, after a sufficient time the combined interference of forward and backward waves becomes stable, and one can identify two well-defined incoming and reflected steady-state waves, arising from the superposition of the infinite transient components travelling on the line.

Note that the wave fronts travel with a phase velocity equal to the speed of light in the medium surrounding the wires. Also, note that the length of the line will affect the interference pattern of the wave superposition, so that lines of different length will result in different voltage and current distributions along the line.

When we study the steady-state voltages and currents in a transmission line, we only need to know the phasors that represent the stable steady-state oscillations at each line location. The phasors provide a snapshot of how values of voltage and current relate to each other in space, at a reference time, in terms of amplitude and phase. The actual time oscillation can be easily recovered, because in steady-state we know that voltage and current are perfectly periodic at each line location, according to the period of the generator.

If the generator provides more than one frequency of oscillation, at steady-state the behavior of each frequency in the spectrum can be studied independently and the total result is obtained by superposition.